ELEMENTARY LECTURES

ON

ELECTRIC DISCHARGES, WAVES AND IMPULSES,

AND

OTHER TRANSIENTS

BY

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LECTURE IX.

OSCILLATIONS OF THE COMPOUND CIRCUIT.

38. The most interesting and most important application of the traveling wave is that of the stationary oscillation of a compound circuit, as industrial circuits are never uniform, but consist of sections of different characteristics, as the generating system, transformer, line, load, etc. Oscillograms of such circuits have been shown in the previous lecture.

If we have a circuit consisting of sections 1, 2, 3 . . . , of the respective lengths (in velocity measure) λ_1, λ_2, λ_3 . . . , this entire circuit, when left to itself, gradually dissipates its stored energy by a transient. As function of the time, this transient must decrease at the same rate u_0 throughout the entire circuit. Thus the time decrement of all the sections must be

$$\epsilon^{-u_0 t}.$$

Every section, however, has a power-dissipation constant, u_1, u_2, u_3 . . . , which represents the rate at which the stored energy of the section would be dissipated by the losses of power in the section,

$$\epsilon^{-u_1 t},\ \epsilon^{-u_2 t},\ \epsilon^{-u_3 t}\ .\ .\ .$$

But since as part of the whole circuit each section must die down at the same rate $\epsilon^{-u_0 t}$, in addition to its power-dissipation decrement $\epsilon^{-u_1 t}$, $\epsilon^{-u_2 t}$ . . . , each section must still have a second time decrement, $\epsilon^{-(u_0-u_1)t}$, $\epsilon^{-(u_0-u_2)t}$ . . . This latter does not represent power dissipation, and thus represents power transfer. That is,

$$\begin{aligned} s_1 &= u_0 - u_1, \\ s_2 &= u_0 - u_2, \\ &\cdot \\ &\cdot \\ &\cdot \end{aligned} \qquad (1)$$

It thus follows that in a compound circuit, if u_0 is the average exponential time decrement of the complete circuit, or the average

power-dissipation constant of the circuit, and u that of any section, this section must have a second exponential time decrement,

$$s = u_0 - u, \qquad (2)$$

which represents power transfer from the section to other sections, or, if s is negative, power received from other sections. The oscillation of every individual section thus is a traveling wave, with a power-transfer constant s.

As u_0 is the average dissipation constant, that is, an average of the power-dissipation constants u of all the sections, and $s = u_0 - u$ the power-transfer constant, some of the s must be positive, some negative.

In any section in which the power-dissipation constant u is less than the average u_0 of the entire circuit, the power-transfer constant s is positive; that is, the wave, passing over this section, increases in intensity, builds up, or in other words, gathers energy, which it carries away from this section into other sections. In any section in which the power-dissipation constant u is greater than the average u_0 of the entire circuit, the power-transfer constant s is negative; that is, the wave, passing over this section, decreases in intensity and thus in energy, or in other words, leaves some of its energy in this section, that is, supplies energy to the section, which energy it brought from the other sections.

By the power-transfer constant s, sections of low energy dissipation supply power to sections of high energy dissipation.

39. Let for instance in Fig. 43 be represented a circuit consisting of step-up transformer, transmission line, and load. (The load, consisting of step-down transformer and its secondary circuit, may for convenience be considered as one circuit section.) Assume now that the circuit is disconnected from the power supply by low-tension switches, at A. This leaves transformer, line, and load as a compound oscillating circuit, consisting of four sections: the high-tension coil of the step-up transformer, the two lines, and the load.

Let then λ_1 = length of line, λ_2 = length of transformer circuit, and λ_3 = length of load circuit, in velocity measure.* If then

* If l_1 = length of circuit section in any measure, and L_0 = inductance, C_0 = capacity per unit of length l_1, then the length of the circuit in velocity measure is $\lambda_1 = \sigma_0 l_1$, where $\sigma_0 = \sqrt{L_0 C_0}$.

Thus, if L = inductance, C = capacity per transformer coil, n = number of transformer coils, for the transformer the unit of length is the coil; hence the

$u_1 = 900$ = power-dissipation constant of the line, $u_2 = 100$ = power-dissipation constant of transformer, and $u_3 = 1600$ = power-dissipation constant of the load, and the respective lengths of the circuit sections are

$$\lambda_1 = 1.5 \times 10^{-3};\ \lambda_2 = 1 \times 10^{-3};\ \lambda_3 = 0.5 \times 10^{-3},$$

it is:

		Line.	Transformer.	Line.	Load.	Sum.
Length:	$\lambda =$	1.5×10^{-3}	1×10^{-3}	1.5×10^{-3}	$.5\times10^{-3}$	4.5×10^{-3}
Power-dissipation constant:	$u =$	900	100	900	1600	
	$u\lambda =$	1.35	.1	1.35	.8	3.6

hence, $u_0 =$ average $u = \dfrac{\Sigma u\lambda}{\Sigma\lambda} = 800$, and:

Power-transfer constant:	$s = u_0 - u =$	-100	$+700$	-100	-800

The transformer thus dissipates power at the rate $u_2 = 100$, while it sends out power into the other sections at the rate of $s_2 = 700$, or seven times as much as it dissipates. That is, it supplies seven-eighths of its stored energy to other sections. The load dissipates power at the rate $u_3 = 1600$, and receives power at the rate $-s = 800$; that is, half of the power which it dissipates is supplied from the other sections, in this case the transformer.

The transmission line dissipates power at the rate $u_1 = 900$, that is, only a little faster than the average power dissipation of the entire circuit, $u_0 = 800$; and the line thus receives power only at the rate $-s = 100$, that is, receives only one-ninth of its power from the transformer; the other eight-ninths come from its stored energy.

The traveling wave passing along the circuit section thus increases or decreases in its power at the rate $\epsilon^{+2s\lambda}$; that is, in the line:

$$p = p_1\epsilon^{-200\lambda}, \text{ the energy of the wave decreases slowly;}$$

in the transformer:

$$p = p_2\epsilon^{+1400\lambda}, \text{ the energy of the wave increases rapidly;}$$

length $l_1 = n$, and the length in velocity measure, $\lambda = \sigma_0 n = n\sqrt{LC}$. Or, if L = inductance, C = capacity of the entire transformer, its length in velocity measure is $\lambda = \sqrt{LC}$.

Thus, the reduction to velocity measure of distance is very simple.

in the load:

$$p = p_3\epsilon^{-1600\lambda}, \text{ the energy of the wave decreases rapidly.}$$

Here the coefficients of p_1, p_2, p_3 must be such that the wave at the beginning of one section has the same value as at the end of the preceding section.

In general, two traveling waves run around the circuit in opposite direction.

Each of the two waves reaches its maximum intensity in this circuit at the point where it leaves the transformer and enters the line, since in the transformer it increases, while in the line it again decreases, in intensity.

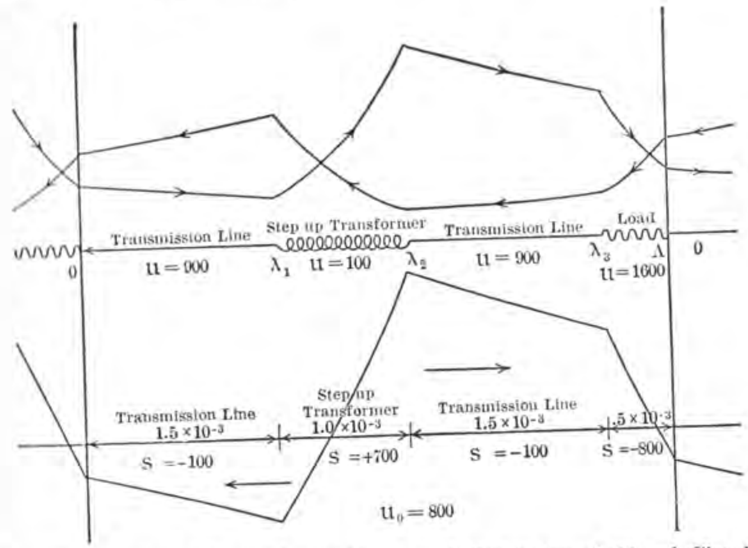

Fig. 54. — Energy Distribution in Compound Oscillation of Closed Circuit; High Line Loss.

Assuming that the maximum value of the one wave is 6, that of the opposite wave 4 megawatts, the power values of the two waves then are plotted in the upper part of Fig. 54, and their difference, that is, the resultant flow of power, in the lower part of Fig. 54. As seen from the latter, there are two power nodes, or points over which no power flows, one in the transformer and one in the load, which no power flows, one in the transformer and one in the load, and the power flows from the transformer over the line into the load; the transformer acts as generator of the power, and of this power a fraction is consumed in the line, the rest supplied to the load.

40. The diagram of this transient power transfer of the system thus is very similar to that of the permanent power transmission by alternating currents: a source of power, a partial consumption in the line, and the rest of the power consumed in the load.

However, this transient power-transfer diagram does not represent the entire power which is being consumed in the circuit, as power is also supplied from the stored energy of the circuit; and the case may thus arise — which cannot exist in a permanent power transmission — that the power dissipation of the line is less than corresponds to its stored energy, and the line also supplies power to the load, that is, acts as generator, and in this case the power would not be a maximum at the transformer terminals, but would still further increase in the line, reaching its maximum at the load terminals. This obviously is possible only with transient power, where the line has a store of energy from which it can draw in supplying power. In permanent condition the line could not add to the power, but must consume, that is, the permanent power-transmission diagram must always be like Fig. 54.

Not so, as seen, with the transient of the stationary oscillation. Assume, for instance, that we reduce the power dissipation in the line by doubling the conductor section, that is, reducing the resistance to one-half. As L thereby also slightly decreases, C increases, and g possibly changes, the change brought about in the constant $u = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$ is not necessarily a reduction to one-half, but depends upon the dimensions of the line. Assuming therefore, that the power-dissipation constant of the line is by the doubling of the line section reduced from $u_1 = 900$ to $u_1 = 500$, this gives the constants:

	Line.	Transformer.	Line.	Load.	Sum.
$\lambda =$	1.5×10^{-3}	1×10^{-3}	1.5×10^{-3}	$.5 \times 10^{-3}$	4.5×10^{-3}
$u =$	500	100	500	1600	
$u\lambda =$	.75	.1	.75	.8	2.4

hence, $u_0 =$ average $u = \frac{\Sigma u\lambda}{\Sigma\lambda} = 533$, and:

$s =$	+33	+133	+33	−1067

That is, the power-transfer constant of the line has become positive, $s_1 = 33$, and the line now assists the transformer in supplying power to the load. Assuming again the values of the two traveling waves, where they leave the transformer (which now are not the maximum values, since the waves still further increase in intensity in passing over the lines), as 6 and 4 megawatts respectively, the power diagram of the two waves, and the power diagram of their resultant, are given in Fig. 55.

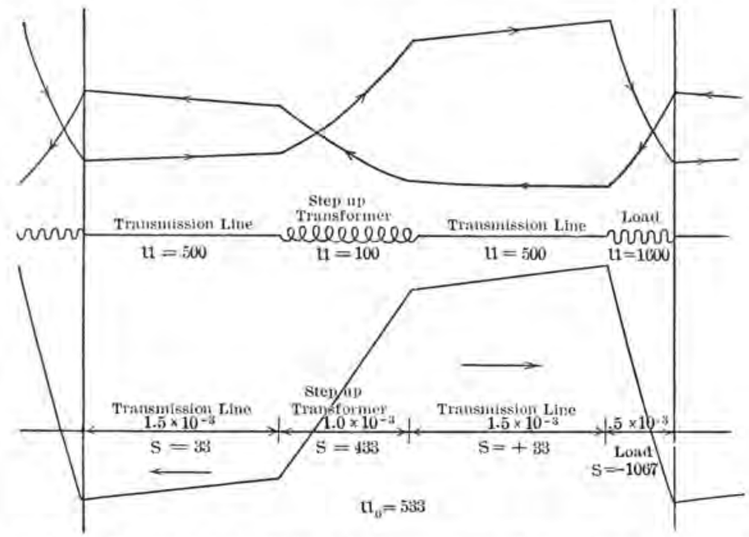

Fig. 55. — Energy Distribution in Compound Oscillation of Closed Circuit; Low Line Loss.

In a closed circuit, as here discussed, the relative intensity of the two component waves of opposite direction is not definite, but depends on the circuit condition at the starting moment of the transient.

In an oscillation of an open compound circuit, the relative intensities of the two component waves are fixed by the condition that at the open ends of the circuit the power transfer must be zero.

As illustration may be considered a circuit comprising the high-potential coil of the step-up transformer, and the two lines, which are assumed as open at the step-down end, as illustrated diagrammatically in Fig. 56.

Choosing the same lengths and the same power-dissipation constants as in the previous illustrations, this gives:

	Line.	Transformer.	Line.	Sum.
$\lambda =$	1.5×10^{-3}	1×10^{-3}	1.5×10^{-3}	4×10^{-3}
$u =$	900	100	900	
$u\lambda =$	1.35	.1	1.35	2.8

hence, $u_0 =$ average $u = \frac{\Sigma u\lambda}{\Sigma\lambda} = 700$, and:

$s =$	-200	$+600$	-200

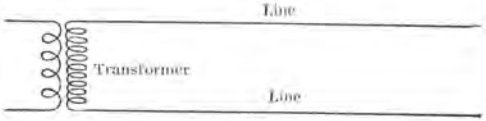

Fig. 56.

The diagram of the power of the two waves of opposite directions, and of the resultant power, is shown in Fig. 57, assuming 6 megawatts as the maximum power of each wave, which is reached at the point where it leaves the transformer.

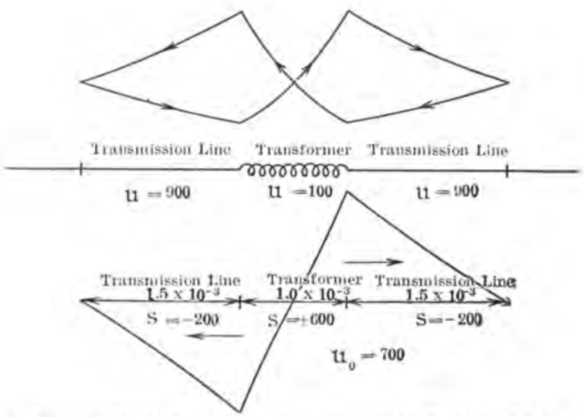

Fig. 57. — Energy Distribution in Compound Oscillation of Open Circuit.

In this case the two waves must be of the same intensity, so as to give 0 as resultant at the open ends of the line. A power node then appears in the center of the transformer.

41. A stationary oscillation of a compound circuit consists of two traveling waves, traversing the circuit in opposite direction, and transferring power between the circuit sections in such a manner

as to give the same rate of energy dissipation in all circuit sections. As the result of this power transfer, the stored energy of the system must be uniformly distributed throughout the entire circuit, and if it is not so in the beginning of the transient, local traveling waves redistribute the energy throughout the oscillating circuit, as stated before. Such local oscillations are usually of very high frequency, but sometimes come within the range of the oscillograph, as in Fig. 47.

During the oscillation of the complex circuit, every circuit element $d\lambda$ (in velocity measure), or every wave length or equal part of the wave length, therefore contains the same amount of stored energy. That is, if e_0 = maximum voltage, i_0 = maximum current, and λ_0 = wave length, the average energy $\frac{e_0 i_0 \lambda_0}{2}$ must be constant throughout the entire circuit. Since, however, in velocity measure, λ_0 is constant and equal to the period T_0 throughout all the sections of the circuit, the product of maximum voltage and of maximum current, $e_0 i_0$, thus must be constant throughout the entire circuit.

The same applies to an ordinary traveling wave or impulse. Since it is the same energy which moves along the circuit at a constant rate, the energy contents for equal sections of the circuit must be the same except for the factor ϵ^{-2ut}, by which the energy decreases with the time, and thus with the distance traversed during this time.

Maximum voltage e_0 and maximum current i_0, however, are related to each other by the condition

$$\frac{e_0}{i_0} = z_0 = \sqrt{\frac{L_0}{C_0}}, \tag{3}$$

and as the relation of L_0 and C_0 is different in the different sections, and that very much so, z_0, and with it the ratio of maximum voltage to maximum current, differ for the different sections of the circuit.

If then e_1 and i_1 are maximum voltage and maximum current respectively of one section, and $z_1 = \sqrt{\frac{L_1}{C_1}}$ is the "natural impedance" of this section, and e_2, i_2, and $z_2 = \sqrt{\frac{L_2}{C_2}}$ are the corresponding values for another section, it is

$$e_2 i_2 = e_1 i_1, \tag{4}$$

and since

$$\frac{e_2}{i_2} = z_2; \quad \frac{e_1}{i_1} = z_1, \tag{5}$$

substituting

$$\left.\begin{aligned} e_2 &= i_2 z_2, \\ e_1 &= i_1 z_1, \end{aligned}\right\} \tag{6}$$

into (4) gives

$$i_2^2 z_2 = i_1^2 z_1,$$

or

$$\frac{i_2}{i_1} = \sqrt{\frac{z_1}{z_2}} = \sqrt[4]{\frac{L_1 C_2}{C_1 L_2}}, \tag{7}$$

and

$$\frac{e_2^2}{z_2} = \frac{e_1^2}{z_1},$$

or

$$\frac{e_2}{e_1} = \sqrt{\frac{z_2}{z_1}} = \sqrt[4]{\frac{L_2 C_1}{C_2 L_1}}. \tag{8}$$

That is, in the same oscillating circuit, the maximum voltages e_0 in the different sections are proportional to, and the maximum currents i_0 inversely proportional to, the square root of the natural impedances z_0 of the sections, that is, to the fourth root of the ratios of inductance to capacity $\frac{L_0}{C_0}$.

At every transition point between successive sections traversed by a traveling wave, as those of an oscillating system, a transformation of voltage and of current occurs, by a transformation ratio which is the square root of the ratio of the natural impedances, $z_0 = \sqrt{\frac{L_0}{C_0}}$, of the two respective sections.

When passing from a section of high capacity and low inductance, that is, low impedance z_0, to a section of low capacity and high inductance, that is, high impedance z_0, as when passing from a transmission line into a transformer, or from a cable into a transmission line, the voltage thus is transformed up, and the current transformed down, and inversely, with a wave passing in opposite direction.

A low-voltage high-current wave in a transmission line thus becomes a high-voltage low-current wave in a transformer, and inversely, and thus, while it may be harmless in the line, may become destructive in the transformer, etc.

42. At the transition point between two successive sections, the current and voltage respectively must be the same in the two sections. Since the maximum values of current and voltage respectively are different in the two sections, the phase angles of the waves of the two sections must be different at the transition point; that is, a change of phase angle occurs at the transition point.

This is illustrated in Fig. 58. Let $z_0 = 200$ in the first section (transmission line), $z_0 = 800$ in the second section (transformer). The transformation ratio between the sections then is $\sqrt{\frac{800}{200}} = 2$; that is, the maximum voltage of the second section is twice, and the maximum current half, that of the first section, and the waves of current and of voltage in the two sections thus may be as illustrated for the voltage in Fig. 58, by $e_1 e_2$.

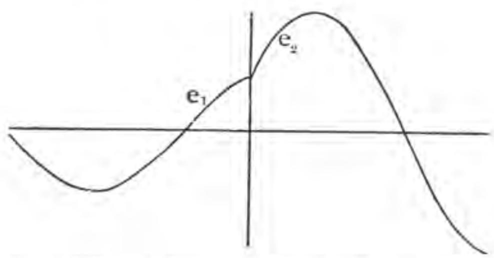

Fig. 58. — Effect of Transition Point on Traveling Wave.

If then e' and i' are the values of voltage and current respectively at the transition point between two sections 1 and 2, and e_1 and i_1 the maximum voltage and maximum current respectively of the first, e_2 and i_2 of the second, section, the voltage phase and current phase at the transition point are, respectively:

For the wave of the first section:

$$\frac{e'}{e_1} = \cos \gamma_1 \quad \text{and} \quad \frac{i'}{i_1} = \cos \delta_1.$$

For the wave of the second section:

$$\frac{e'}{e_2} = \cos \gamma_2 \quad \text{and} \quad \frac{i'}{i_2} = \cos \delta_2. \tag{9}$$

Dividing the two pairs of equations of (9) gives

$$\left.\begin{aligned} \frac{\cos\gamma_2}{\cos\gamma_1} &= \frac{e_1}{e_2} = \sqrt{\frac{z_1}{z_2}}, \\ \frac{\cos\delta_2}{\cos\delta_1} &= \frac{i_1}{i_2} = \sqrt{\frac{z_2}{z_1}}, \end{aligned}\right\} \tag{10}$$

hence, multiplied,

$$\frac{\cos\gamma_2}{\cos\gamma_1} \times \frac{\cos\delta_2}{\cos\delta_1} = 1,$$

or

$$\frac{\cos\gamma_2}{\cos\gamma_1} = \frac{\cos\delta_1}{\cos\delta_2}, \tag{11}$$

or

$$\cos\gamma_1\cos\delta_1 = \cos\gamma_2\cos\delta_2;$$

that is, the ratio of the cosines of the current phases at the transition point is the reciprocal of the ratio of the cosines of the voltage phases at this point.

Since at the transition point between two sections the voltage and current change, from e_1, i_1 to e_2, i_2, by the transformation ratio $\sqrt{\frac{z_2}{z_1}}$, this change can also be represented as a partial reflection. That is, the current i_1 can be considered as consisting of a component i_2, which passes over the transition point, is "transmitted" current, and a component $i_1' = i_1 - i_2$, which is "reflected" current, etc. The greater then the change of circuit constants at the transition point, the greater is the difference between the currents and voltages of the two sections; that is, the more of current and voltage are reflected, the less transmitted, and if the change of constants is very great, as when entering from a transmission line a reactance of very low capacity, almost all the current is reflected, and very little passes into and through the reactance, but a high voltage is produced in the reactance.